# 利用正射影與加減消去法來求兩歪斜線的公垂線段的兩端點座標

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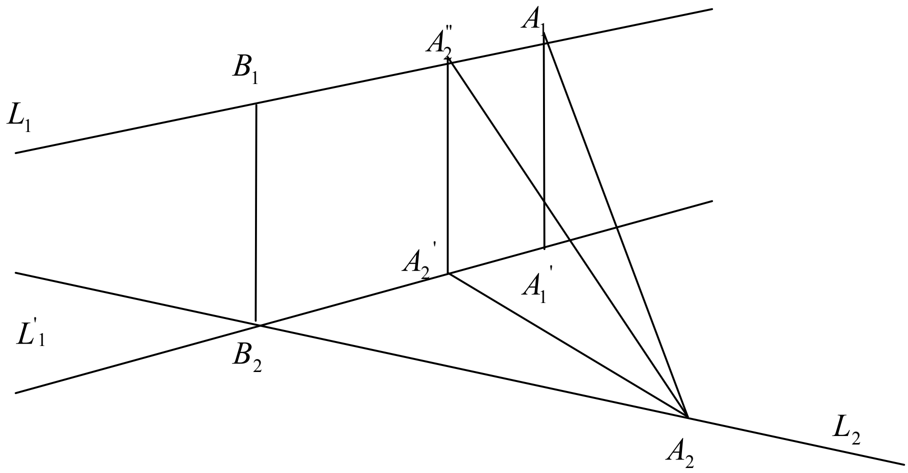
研究目的：試圖利用正射影與加減消去法來求兩歪斜線的公垂線段的兩端點座標。

研究過程：




如上圖所示，已知空間直角座標系中，$O$爲原點，兩歪斜線$L_1$與$L_2$分別通過點$A_1$、點$A_2$，$L_1$與$L_2$的方向向量分別爲$\vec{d}_1$與$\vec{d}_2$ ，$\vec{d}_1$不平行$\vec{d}_2$ ，四邊形$A_1A_1'B_2B_1$爲矩形，四邊形$A_2''A_2'B_2B_1$爲矩形，$\angle A_2A_2'A_1'=90^\circ,\angle A_2A_2''A_1=90^\circ$ ，$\overline{B_1B_2}\perp L_1$與$L_2$ ，求點$B_1$及點$B_2$的座標。

解: 設$\overrightarrow{A_1B_1}=\overrightarrow{A_1'B_2}=t_1\vec{d}_1$ ，$\overrightarrow{A_2B_2}=t_2\vec{d}_2$ 。利用正射影概念，

$$\because \overrightarrow{A_2'A_1'}=\overrightarrow{A_2''A_1}\ \therefore \frac{\overrightarrow{A_2A_1'}\cdot\vec{d}_1}{\left|\vec{d}_1\right|^2}\vec{d}_1=\frac{(\overrightarrow{A_2B_2}-\overrightarrow{A_1'B_2})\cdot\vec{d}_1}{\left|\vec{d}_1\right|^2}\vec{d}_1=\frac{(t_2\vec{d}_2-t_1\vec{d}_1)\cdot\vec{d}_1}{\left|\vec{d}_1\right|^2}\vec{d}_1=\frac{\overrightarrow{A_2A_1}\cdot\vec{d}_1}{\left|\vec{d}_1\right|^2}\vec{d}_1$$ ，

$\Rightarrow t_2(\vec{d}_1\cdot\vec{d}_2)-t_1(\vec{d}_1\cdot\vec{d}_1)=\overrightarrow{A_2A_1}\cdot\vec{d}_1......(a)$ 。　　同理可得　$t_1(\vec{d}_1\cdot\vec{d}_2)-t_2(\vec{d}_2\cdot\vec{d}_2)=\overrightarrow{A_1A_2}\cdot\vec{d}_2......(b)$

$$(a)\times(-\vec{d}_2\cdot\vec{d}_2)+(b)\times(-\vec{d}_1\cdot\vec{d}_2)\Rightarrow t_1[(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)]=[(\overrightarrow{A_1A_2}\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\overrightarrow{A_1A_2}\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)]$$

$$(a)\times(-\vec{d}_1\cdot\vec{d}_2)+(b)\times(-\vec{d}_1\cdot\vec{d}_1)\Rightarrow t_2[(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)]=[(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\overrightarrow{A_2A_1})-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\overrightarrow{A_2A_1})]$$

$\because \vec{d}_1$不平行$\vec{d}_2$ $\therefore$由$\vec{d}_1$與$\vec{d}_2$所形成平行四邊形的面積平方$=[(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)]\neq 0$

因此$\begin{cases} t_1=\dfrac{(\overrightarrow{A_1A_2}\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\overrightarrow{A_1A_2}\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)}{(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\vec{d}_2)} \\ t_2=\dfrac{(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\overrightarrow{A_2A_1})-(\vec{d}_1\cdot\vec{d}_2)(\vec{d}_1\cdot\overrightarrow{A_2A_1})}{(\vec{d}_1\cdot\vec{d}_1)(\vec{d}_2\cdot\vec{d}_2)-(\vec{d}_1\cdot\vec{d}_2)\cdot(\vec{d}_1\cdot\vec{d}_2)} \end{cases}$ ，解得$\begin{cases} \overrightarrow{OB_1}=\overrightarrow{OA_1}+t_1\vec{d}_1 \\ \overrightarrow{OB_2}=\overrightarrow{OA_2}+t_2\vec{d}_2 \end{cases}$ 。