兩歪斜線的公垂線段的兩端點座標的公式解法比較分析

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研究目的：試圖比較分析兩歪斜線的公垂線段的兩端點座標的公式解法。

研究過程：

已知空間直角座標系中，O爲原點，兩歪斜線L_1與L_2分別通過點$A_1\ (x_1,y_1,z_1)$、點$A_2\ (x_2,y_2,z_2)$，L_1與L_2的方向向量分別爲$\vec{d}_1=(l_1,m_1,n_1)$與$\vec{d}_2=(l_2,m_2,n_2)$，試求L_1與公垂線的交點B_1及L_2與公垂線的交點B_2的座標。

一、建立二元一次聯立方程組(方法一)：(投稿中研院數學傳播季刊，民國九十九年 10 月 26 日，編號 3571，審稿中。)

1. $\because B_1\in L_1, \therefore B_1=(x_1+l_1t_1, y_1+m_1t_1, z_1+n_1t_1)$，$t_1\in R$，

$\because B_2\in L_2, \therefore B_2=(x_2+l_2t_2, y_2+m_2t_2, z_2+n_2t_2)$，$t_2\in R$。

$\Rightarrow \overrightarrow{B_1B_2}=(x_2-x_1+l_2t_2-l_1t_1, y_2-y_1+m_2t_2-m_1t_1, z_2-z_1+n_2t_2-n_1t_1)$。

2. $\because \overrightarrow{B_1B_2}\perp\vec{d}_1$ 且 $\overrightarrow{B_1B_2}\perp\vec{d}_2$，

$\therefore \overrightarrow{B_1B_2}\cdot\vec{d}_1=0$ 且 $\overrightarrow{B_1B_2}\cdot\vec{d}_2=0$。

3. $$\begin{cases}0=\overrightarrow{B_1B_2}\cdot\vec{d}_1=(x_2-x_1+l_2t_2-l_1t_1)l_1+(y_2-y_1+m_2t_2-m_1t_1)m_1+(z_2-z_1+n_2t_2-n_1t_1)n_1\\0=\overrightarrow{B_1B_2}\cdot\vec{d}_2=(x_2-x_1+l_2t_2-l_1t_1)l_2+(y_2-y_1+m_2t_2-m_1t_1)m_2+(z_2-z_1+n_2t_2-n_1t_1)n_2\end{cases},$$

$$\Rightarrow\begin{cases}0=[(x_2-x_1)l_1+(y_2-y_1)m_1+(z_2-z_1)n_1]+t_2(l_1l_2+m_1m_2+n_1n_2)-t_1(l_1^2+m_1^2+n_1^2)\\0=[(x_2-x_1)l_2+(y_2-y_1)m_2+(z_2-z_1)n_2]+t_2(l_2^2+m_2^2+n_2^2)-t_1(l_1l_2+m_1m_2+n_1n_2)\end{cases},$$

$$\Rightarrow\begin{cases}0=\overrightarrow{A_1A_2}\cdot\vec{d}_1+t_2(\vec{d}_1\cdot\vec{d}_2)-t_1(\vec{d}_1\cdot\vec{d}_1)\\0=\overrightarrow{A_1A_2}\cdot\vec{d}_2+t_2(\vec{d}_2\cdot\vec{d}_2)-t_1(\vec{d}_1\cdot\vec{d}_2)\end{cases},$$

$$\Rightarrow\begin{cases}t_1(\vec{d}_1\cdot\vec{d}_1)-t_2(\vec{d}_1\cdot\vec{d}_2)=\overrightarrow{A_1A_2}\cdot\vec{d}_1\\t_1(\vec{d}_1\cdot\vec{d}_2)-t_2(\vec{d}_2\cdot\vec{d}_2)=\overrightarrow{A_1A_2}\cdot\vec{d}_2\end{cases}。$$

建立二元一次聯立方程組(方法二)：(本次 MTS2010 發表的文章。)

4.設 $\overrightarrow{A_1B_1}=t_1\vec{d}_1$，$\overrightarrow{A_2B_2}=t_2\vec{d}_2$，

則 $\overrightarrow{B_1B_2}=\overrightarrow{B_1A_1}+\overrightarrow{A_1A_2}+\overrightarrow{A_2B_2}=-t_1\vec{d}_1+\overrightarrow{A_1A_2}+t_2\vec{d}_2$。

5.$\because \overrightarrow{B_1B_2}\perp\vec{d}_1$ 且 $\overrightarrow{B_1B_2}\perp\vec{d}_2$。

$\therefore \overrightarrow{B_1B_2}\cdot\vec{d}_1=0$ 且 $\overrightarrow{B_1B_2}\cdot\vec{d}_2=0$。

6. $$\begin{cases}0=\overrightarrow{B_1B_2}\cdot\vec{d}_1=-t_1(\vec{d}_1\cdot\vec{d}_1)+\overrightarrow{A_1A_2}\cdot\vec{d}_1+t_2(\vec{d}_1\cdot\vec{d}_2)\\0=\overrightarrow{B_1B_2}\cdot\vec{d}_2=-t_1(\vec{d}_1\cdot\vec{d}_2)+\overrightarrow{A_1A_2}\cdot\vec{d}_2+t_2(\vec{d}_2\cdot\vec{d}_2)\end{cases},$$

$$\Rightarrow\begin{cases}t_1(\vec{d}_1\cdot\vec{d}_1)-t_2(\vec{d}_1\cdot\vec{d}_2)=\overrightarrow{A_1A_2}\cdot\vec{d}_1\\t_1(\vec{d}_1\cdot\vec{d}_2)-t_2(\vec{d}_2\cdot\vec{d}_2)=\overrightarrow{A_1A_2}\cdot\vec{d}_2\end{cases}。$$

二、利用克拉瑪公式求解：

1.設$\vec{d}_1$與$\vec{d}_2$的夾角爲θ，$\because$ $\vec{d}_1$與$\vec{d}_2$不平行$\therefore \sin\theta \neq 0$。

2. $\Delta = \begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & -\vec{d}_1\cdot\vec{d}_2 \\ \vec{d}_1\cdot\vec{d}_2 & -\vec{d}_2\cdot\vec{d}_2 \end{vmatrix} = -[|\vec{d}_1|^2|\vec{d}_2|^2-(\vec{d}_1\cdot\vec{d}_2)^2] = -|\vec{d}_1|^2|\vec{d}_2|^2\sin^2\theta \neq 0$，

由克拉瑪公式得知(t_1,t_2)恰有一組解。

$$t_1 = \frac{\begin{vmatrix} \overrightarrow{A_1A_2}\cdot\vec{d}_1 & -\vec{d}_1\cdot\vec{d}_2 \\ \overrightarrow{A_1A_2}\cdot\vec{d}_2 & -\vec{d}_2\cdot\vec{d}_2 \end{vmatrix}}{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & -\vec{d}_1\cdot\vec{d}_2 \\ \vec{d}_1\cdot\vec{d}_2 & -\vec{d}_2\cdot\vec{d}_2 \end{vmatrix}} = \frac{\begin{vmatrix} \overrightarrow{A_1A_2}\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2 \\ \overrightarrow{A_1A_2}\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2 \end{vmatrix}}{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2 \\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2 \end{vmatrix}}，$$

$$t_2 = \frac{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\overrightarrow{A_1A_2} \\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\overrightarrow{A_1A_2} \end{vmatrix}}{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & -\vec{d}_1\cdot\vec{d}_2 \\ \vec{d}_1\cdot\vec{d}_2 & -\vec{d}_2\cdot\vec{d}_2 \end{vmatrix}} = \frac{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\overrightarrow{A_2A_1} \\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\overrightarrow{A_2A_1} \end{vmatrix}}{\begin{vmatrix} \vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2 \\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2 \end{vmatrix}}。$$

三、建立一元一次方程式(方法一)：(已刊登在中研院數學傳播季刊，民國 98 年 12 月。)

1. 過 L_1 與 B_2 的平面 E_1 的法向量 $\vec{n}_1 \parallel \left|\vec{d}_1\right|^2 \vec{d}_2 - (\vec{d}_1 \cdot \vec{d}_2)\vec{d}_1$ ，且

$$\overrightarrow{A_2B_2} \cdot \vec{n}_1 = \overrightarrow{A_2A_1} \cdot \vec{n}_1$$

$$\Rightarrow t_2\vec{d}_2 \cdot \vec{n}_1 = \overrightarrow{A_2A_1} \cdot \vec{n}_1 \text{ ，}$$

$$\text{解得 } t_2 = \frac{\overrightarrow{A_2A_1} \cdot \vec{n}_1}{\vec{d}_2 \cdot \vec{n}_1} = \frac{\left|\vec{d}_1\right|^2 (\vec{d}_2 \cdot \overrightarrow{A_2A_1}) - (\vec{d}_1 \cdot \vec{d}_2)(\vec{d}_1 \cdot \overrightarrow{A_2A_1})}{\left|\vec{d}_1\right|^2 \left|\vec{d}_2\right|^2 - (\vec{d}_1 \cdot \vec{d}_2)^2}$$

$$= \frac{\begin{vmatrix} \vec{d}_1 \cdot \vec{d}_1 & \vec{d}_1 \cdot \overrightarrow{A_2A_1} \\ \vec{d}_1 \cdot \vec{d}_2 & \vec{d}_2 \cdot \overrightarrow{A_2A_1} \end{vmatrix}}{\begin{vmatrix} \vec{d}_1 \cdot \vec{d}_1 & \vec{d}_1 \cdot \vec{d}_2 \\ \vec{d}_1 \cdot \vec{d}_2 & \vec{d}_2 \cdot \vec{d}_2 \end{vmatrix}} \text{ 。}$$

2. 過 L_2 與 B_1 的平面 E_2 的法向量 $\vec{n}_2 \parallel \left|\vec{d}_2\right|^2 \vec{d}_1 - (\vec{d}_1 \cdot \vec{d}_2)\vec{d}_2$ 。

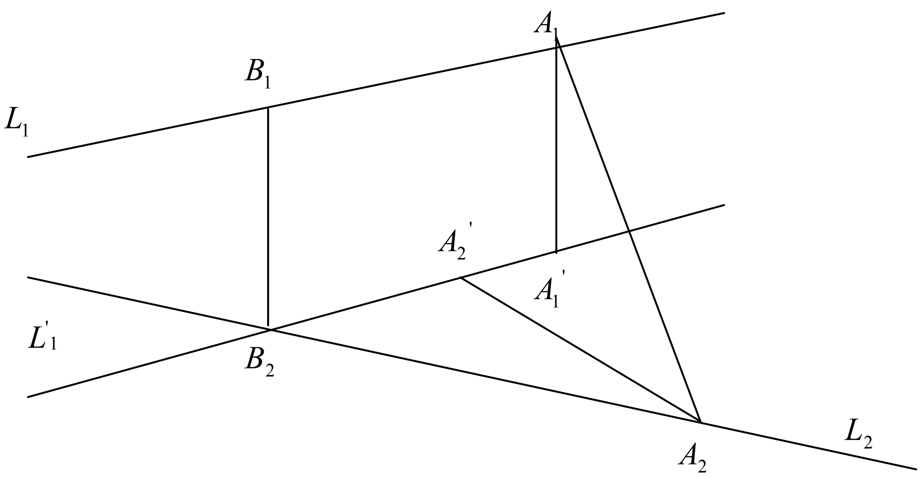
$$\overrightarrow{A_1B_1} \cdot \vec{n}_2 = \overrightarrow{A_1A_2} \cdot \vec{n}_2$$

$$\Rightarrow t_1\vec{d}_1 \cdot \vec{n}_2 = \overrightarrow{A_1A_2} \cdot \vec{n}_2 \text{ ，}$$

$$\text{解得 } t_1 = \frac{\overrightarrow{A_1A_2} \cdot \vec{n}_2}{\vec{d}_1 \cdot \vec{n}_2} = \frac{\left|\vec{d}_2\right|^2 (\vec{d}_1 \cdot \overrightarrow{A_1A_2}) - (\vec{d}_1 \cdot \vec{d}_2)(\vec{d}_2 \cdot \overrightarrow{A_1A_2})}{\left|\vec{d}_1\right|^2 \left|\vec{d}_2\right|^2 - (\vec{d}_1 \cdot \vec{d}_2)^2}$$

$$= \frac{\begin{vmatrix} \overrightarrow{A_1A_2} \cdot \vec{d}_1 & \vec{d}_1 \cdot \vec{d}_2 \\ \overrightarrow{A_1A_2} \cdot \vec{d}_2 & \vec{d}_2 \cdot \vec{d}_2 \end{vmatrix}}{\begin{vmatrix} \vec{d}_1 \cdot \vec{d}_1 & \vec{d}_1 \cdot \vec{d}_2 \\ \vec{d}_1 \cdot \vec{d}_2 & \vec{d}_2 \cdot \vec{d}_2 \end{vmatrix}} \text{ 。}$$

建立一元一次方程式(方法二)：(已刊登在龍騰數亦優民國98年4月)



如上圖所示，已知空間直角座標系中，O爲原點，兩歪斜線L_1與L_2分別通過點A_1、點A_2，L_1與L_2的方向向量分別爲$\vec{d}_1$與$\vec{d}_2$，四邊形$A_1A_1^{'}B_2B_1$爲矩形，$\angle A_2A_2^{'}B_2 = 90^\circ$，$\overline{B_1B_2} \perp L_1$與$L_2$，試求$\overline{B_1B_2}$的長度、點$B_1$及點$B_2$的座標。

解：(1)因爲四邊形$A_1A_1^{'}B_2B_1$爲矩形，利用正射影及外積的概念，

$$\overrightarrow{B_2B_1} = \overrightarrow{A_1^{'}A_1} = \frac{\overrightarrow{A_2A_1} \bullet (\vec{d}_1 \times \vec{d}_2)}{\left|\vec{d}_1 \times \vec{d}_2\right|^2}(\vec{d}_1 \times \vec{d}_2)\text{，得}\ \overline{B_1B_2} = \left|\frac{\overrightarrow{A_2A_1} \bullet (\vec{d}_1 \times \vec{d}_2)}{\left|\vec{d}_1 \times \vec{d}_2\right|^2}(\vec{d}_1 \times \vec{d}_2)\right|\text{。}$$

(2)因爲 $\overrightarrow{A_2A_2^{'}} \perp \overrightarrow{B_2A_1} \Rightarrow \overrightarrow{A_2A_2^{'}} \bullet \overrightarrow{B_2A_1} = 0$，

又 $\overrightarrow{A_2A_2^{'}} \parallel \vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)$ 且 $\overrightarrow{B_2A_1} = \overrightarrow{A_2A_1} - \overrightarrow{A_2B_2} = \overrightarrow{A_2A_1} - t_2\vec{d}_2$，

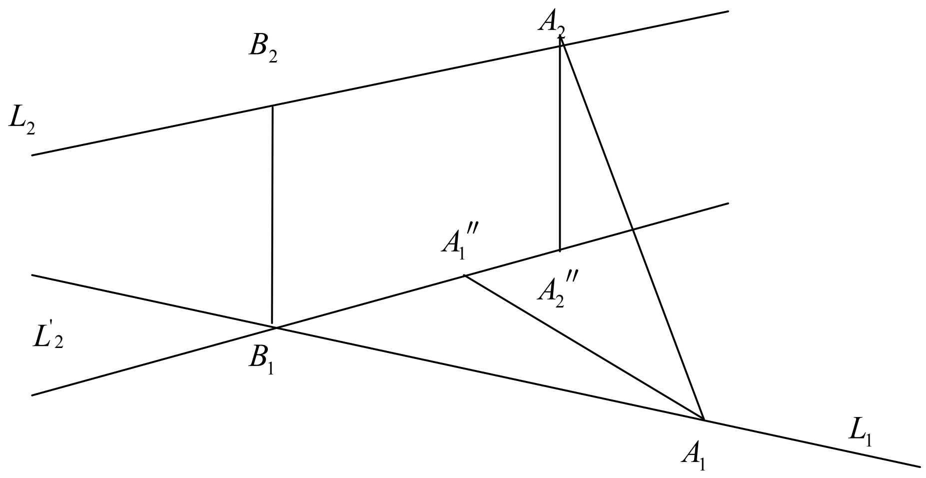
$$\overrightarrow{A_2A_2^{'}} \bullet \overrightarrow{B_2A_1} = 0 \Rightarrow [\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)] \bullet (\overrightarrow{A_2A_1} - t_2\vec{d}_2) = 0\text{，}$$

解得 $t_2 = \dfrac{\overrightarrow{A_2A_1} \bullet [\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)]}{\vec{d}_2 \bullet [\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)]}$

因此

$$\overrightarrow{OB_2} = \overrightarrow{OA_2} + t_2\vec{d}_2\text{，}\overrightarrow{OB_1} = \overrightarrow{OB_2} + \overrightarrow{B_2B_1} = \overrightarrow{OB_2} + \frac{\overrightarrow{A_2A_1} \bullet (\vec{d}_1 \times \vec{d}_2)}{\left|\vec{d}_1 \times \vec{d}_2\right|^2}(\vec{d}_1 \times \vec{d}_2)\text{。}$$

利用同樣的推理方法求得$\overrightarrow{OB_1}=\overrightarrow{OA_1}+\dfrac{\overrightarrow{A_1A_2}\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}{\vec{d}_1\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}\vec{d}_1$ 。



如上圖所示，已知空間直角座標系中，O爲原點，兩歪斜線L_1與L_2分別通過點A_1、點A_2，L_1與L_2的方向向量分別爲$\vec{d}_1$與$\vec{d}_2$，四邊形$A_2A_2''B_1B_2$爲矩形，$\angle A_1A_1''B_1=90°$，$\overline{B_1B_2}\perp L_1$與$L_2$，試求點$B_1$的座標。

解：$\because \overrightarrow{A_1A_1''}\perp\overrightarrow{B_1A_2}\Rightarrow\overrightarrow{A_1A_1''}\cdot\overrightarrow{B_1A_2}=0$ ，

又　$\overrightarrow{A_1A_1''}\parallel\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)$　且　$\overrightarrow{B_1A_2}=\overrightarrow{A_1A_2}-\overrightarrow{A_1B_1}=\overrightarrow{A_1A_2}-t_1\vec{d}_1$ ，

$$\overrightarrow{A_1A_1''}\cdot\overrightarrow{B_1A_2}=0\Rightarrow[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]\cdot(\overrightarrow{A_1A_2}-t_1\vec{d}_1)=0\text{ ，}$$

解得　$t_1=\dfrac{\overrightarrow{A_1A_2}\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}{\vec{d}_1\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}$

因此

$$\overrightarrow{OB_1}=\overrightarrow{OA_1}+t_1\vec{d}_1=\overrightarrow{OA_1}+\frac{\overrightarrow{A_1A_2}\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}{\vec{d}_1\cdot[\vec{d}_2\times(\vec{d}_1\times\vec{d}_2)]}\vec{d}_1\text{ 。}$$

3. 過L_1與B_2的平面E_1的法向量$\vec{n}_1 \parallel \vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)$，且

$$\overrightarrow{A_2B_2} \cdot \vec{n}_1 = \overrightarrow{A_2A_1} \cdot \vec{n}_1$$

$$t_2\vec{d}_2 \cdot \vec{n}_1 = \overrightarrow{A_2A_1} \cdot \vec{n}_1 \text{，}$$

解得 $$t_2 = \frac{\overrightarrow{A_2A_1} \cdot [\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)]}{\vec{d}_2 \cdot [\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)]} \text{。}$$

4. 過L_2與B_1的平面E_2的法向量$\vec{n}_2 \parallel \vec{d}_2 \times (\vec{d}_1 \times \vec{d}_2)$。

$$\overrightarrow{A_1B_1} \cdot \vec{n}_2 = \overrightarrow{A_1A_2} \cdot \vec{n}_2$$

$$t_1\vec{d}_1 \cdot \vec{n}_2 = \overrightarrow{A_1A_2} \cdot \vec{n}_2 \text{，}$$

解得 $$t_1 = \frac{\overrightarrow{A_1A_2} \cdot [\vec{d}_2 \times (\vec{d}_1 \times \vec{d}_2)]}{\vec{d}_1 \cdot [\vec{d}_2 \times (\vec{d}_1 \times \vec{d}_2)]} \text{。}$$

建立一元一次方程式(方法三)：(民國 99 年 2 月已通過審稿，排隊登刊在數學傳播季刊：向量三重積)

利用向量三重積$\vec{\alpha}\times(\vec{\beta}\times\vec{\gamma})=(\vec{\alpha}\cdot\vec{\gamma})\vec{\beta}-(\vec{\alpha}\cdot\vec{\beta})\vec{\gamma}$的恆等式。

一、已知$\vec{\alpha}=(l_1,m_1,n_1)$，$\vec{\beta}=(l_2,m_2,n_2)$，$\vec{\gamma}=(l_3,m_3,n_3)$，

則$\vec{\alpha}\times(\vec{\beta}\times\vec{\gamma})=(\vec{\alpha}\cdot\vec{\gamma})\vec{\beta}-(\vec{\alpha}\cdot\vec{\beta})\vec{\gamma}$。

證明：$\because\ \vec{\beta}\times\vec{\gamma}=(m_2n_3-m_3n_2,n_2l_3-n_3l_2,l_2m_3-l_3m_2)$，

$$\begin{aligned}\therefore\ \vec{\alpha}\times(\vec{\beta}\times\vec{\gamma})=(&m_1(l_2m_3-l_3m_2)-n_1(n_2l_3-n_3l_2),\\ &n_1(m_2n_3-m_3n_2)-l_1(l_2m_3-l_3m_2),\\ &l_1(n_2l_3-n_3l_2)-m_1(m_2n_3-m_3n_2))\\ =((&\vec{\alpha}\cdot\vec{\gamma})l_2-(\vec{\alpha}\cdot\vec{\beta})l_3,(\vec{\alpha}\cdot\vec{\gamma})m_2-(\vec{\alpha}\cdot\vec{\beta})m_3,(\vec{\alpha}\cdot\vec{\gamma})n_2-(\vec{\alpha}\cdot\vec{\beta})n_3)\\ =(&\vec{\alpha}\cdot\vec{\gamma})(l_2,m_2,n_2)-(\vec{\alpha}\cdot\vec{\beta})(l_3,m_3,n_3)\\ =(&\vec{\alpha}\cdot\vec{\gamma})\vec{\beta}-(\vec{\alpha}\cdot\vec{\beta})\vec{\gamma}\end{aligned}$$

，得證。

5. 過L_1與B_2的平面E_1的法向量

$$\vec{n}_1 \parallel \vec{d}_1\times(\vec{d}_1\times\vec{d}_2)=(\vec{d}_1\cdot\vec{d}_2)\vec{d}_1-(\vec{d}_1\cdot\vec{d}_1)\vec{d}_2=-[\left|\vec{d}_1\right|^2\vec{d}_2-(\vec{d}_1\cdot\vec{d}_2)\vec{d}_1]$$ 。

6. 過L_2與B_1的平面E_2的法向量

$$\vec{n}_2 \parallel \vec{d}_2\times(\vec{d}_1\times\vec{d}_2)=\left|\vec{d}_2\right|^2\vec{d}_1-(\vec{d}_1\cdot\vec{d}_2)\vec{d}_2$$ 。

四、結論：

1. $\overrightarrow{OB_1}=\overrightarrow{OA_1}+\overrightarrow{A_1B_1}=\overrightarrow{OA_1}+t_1\vec{d}_1=\overrightarrow{OA_1}+\dfrac{\begin{vmatrix}\overrightarrow{A_1A_2}\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2\\ \overrightarrow{A_1A_2}\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2\end{vmatrix}}{\begin{vmatrix}\vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2\\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2\end{vmatrix}}\vec{d}_1$，

2. $\overrightarrow{OB_2}=\overrightarrow{OA_2}+\overrightarrow{A_2B_2}=\overrightarrow{OA_2}+t_2\vec{d}_2=\overrightarrow{OA_2}+\dfrac{\begin{vmatrix}\vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\overrightarrow{A_2A_1}\\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\overrightarrow{A_2A_1}\end{vmatrix}}{\begin{vmatrix}\vec{d}_1\cdot\vec{d}_1 & \vec{d}_1\cdot\vec{d}_2\\ \vec{d}_1\cdot\vec{d}_2 & \vec{d}_2\cdot\vec{d}_2\end{vmatrix}}\vec{d}_2$。

五、應用：

空間二歪斜線：$L_1:\dfrac{x-11}{4}=\dfrac{y+5}{-3}=\dfrac{z+7}{-1}, L_2:\dfrac{x+5}{3}=\dfrac{y-4}{-4}=\dfrac{z-6}{-2}$ ，

$\overrightarrow{d_1}=(4,-3-1),\overrightarrow{d_2}=(3,-4,-2),\overrightarrow{OA_1}=(11,-5,-7),\overrightarrow{OA_2}=(-5,4,6)$ ，O 爲空間

直角坐標系的原點，$\overrightarrow{A_1A_2}=(-16,9,13)$ 。試求：

(1) L_1 與公垂線的交點 B_1 。
(2) L_2 與公垂線的交點 B_2 。
解：

$$t_1=\frac{\begin{vmatrix}-104 & 26\\ -110 & 29\end{vmatrix}}{\begin{vmatrix}26 & 26\\ 26 & 29\end{vmatrix}}=\frac{\begin{vmatrix}-52 & 26\\ -52 & 29\end{vmatrix}}{\begin{vmatrix}26 & 26\\ 26 & 29\end{vmatrix}}=-2,t_2=\frac{\begin{vmatrix}26 & 104\\ 26 & 110\end{vmatrix}}{\begin{vmatrix}26 & 26\\ 26 & 29\end{vmatrix}}=\frac{\begin{vmatrix}26 & 52\\ 26 & 58\end{vmatrix}}{\begin{vmatrix}26 & 26\\ 26 & 29\end{vmatrix}}=2$$

$\overrightarrow{OB_1}=(11,-5,-7)+(-2)\cdot(4,-3,-1)=(3,1,-5)$ ，

$\overrightarrow{OB_2}=(-5,4,6)+2\cdot(3,-4,-2)=(1,-4,2)$ 。

註：本文所有的射線符號” $\rightarrow$ ”都代表向量符號。

六、參考文獻：

1. 李維昌，”利用正射影及外積的概念求兩歪斜線的公垂線段的距離及兩端點座標”， 龍騰數亦優，第 9 刊，第 17 頁，民國 98 年 4 月 30 日。
2. 李維昌，”利用平面的法向量來求兩歪斜線的公垂線段的兩端點座標”， 數學傳播，第 33 卷，第 4 期，第 63~66 頁，民國 98 年 12 月。

對數換底公式的創意直觀證法

◎宜蘭高中 李維昌

若 a＞0，b＞0，c＞0，而且 $a \neq 1, b \neq 1$，那麼

$$\log_b c = \frac{\log_a c}{\log_a b}$$

傳統證法：(高中課本三民版，作者:楊維哲，蔡聰明，吳隆盛。)(民國 89 年 2 月)

若 $\log_a b = x, \log_b c = y$，

意思是 $b = a^x, c = b^y$，

於是 $c = b^y = (a^x)^y = a^{xy}$，

因此 $\log_a c = xy$，

因而 $\dfrac{\log_a c}{\log_a b} = \dfrac{xy}{x} = y = \log_b c$。

創意直觀證法：(作者:李維昌。)(民國 97 年 2 月)

$$\log_b c = \log_b (a^{\log_a b})^{\frac{\log_a c}{\log_a b}} = \frac{\log_a c}{\log_a b}。$$